Journal of Applied Mechanics and Technical Physics, Vol. 46, No. 3, pp. 423–429, 2005

EFFECT OF A PRECOMPRESSED SPRING ON THE DISCONTINUITY ZONE AROUND A CYLINDRICAL CAVITY

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UDC 622.241.54:539.3

This paper discusses the use of a cylindrical spring to increase the stability of a tunnel of circular cross section.

Key words: *stress state, discontinuity zones, stability of a mine tunnel, support, cylindrical spring.*

In designing and constructing mine tunnels, the most important is the problem of their stability. A tunnel is considered stable if discontinuity zones are not formed in the surrounding massif behind its contour (or these zones small). If a tunnel is constructed at a shallow depth and in fairly strong rock, discontinuity zones, as a rule, do not arise behind its contour. If the rock is weak, the dimensions of the discontinuity zones can be rather large and the tunnel can lose stability, resulting in rock caving. The stability of a tunnel depends on the stress state of the rock massif in the vicinity of the tunnel and the strength properties of the rock.

The problem of the stress state around a tunnel is formulated as follows [1]: an infinite elastic massif is subjected to the stress $\sigma_{33}^\infty = \gamma H$ along the x_3 axis and by the stress $\sigma_{11}^\infty = \sigma_{22}^\infty = \Lambda\gamma H$ in the horizontal direction along the x_1 and x_2 axes; here Λ is the lateral pressure factor; γ is the bulk density of the rock of the massif, and H is the tunnel depth. The specified tunnel is modeled by a cavity inside the massif, whose surface (or any of its part) is subjected from inside to the forces F produced by the reaction of the support. The formulated problem is solved using the method of boundary integral equations, which consists of the following [2]. A compensating load of intensity a is applied to the surface of the cavity. At each point of the cavity, the total stresses from the action of the external and compensating loads should satisfy the conditions on the surface. The stresses from the compensating load are determined by integrating Calvin's solution over the cavity surface. As a result, the conditions on the surface are reduced to the integral equation [2]

$$\frac{1}{2}\,a_q(Q_0) - \iint\limits_O \Phi_{qm}(Q_0, M_0)a_m(M_0)\,dO_{M_0} = n_q(Q_0)\sigma_{qq}^\infty - F_q(Q_0), \qquad (1)$$

where $\Phi_{qm}(Q_0, M_0)$ is Green's tensor, $F_q(Q_0)$ is the reactive response of the support referred to γH, σ_{qq}^∞ is the stress tensor at infinity, O is the surface area of the cavity, n_q and n_m are the unit vectors of the outward (to the cavity surface) normals at the points Q_0 and M_0.

Equation (1) was solved numerically subject to the condition that the tunnel has a circular cross section and has no support $[F_q(Q_0) = 0]$. The discontinuity zone around the tunnel is defined as the set of points at which rock breaking occurred according to Mohr's strength criterion:

$$\bar\tau_\nu = \bar\sigma_\nu \tan\varphi + \bar K.$$

Here $\bar\tau_\nu$ and $\bar\sigma_\nu$ are the dimensionless shear and normal stresses referred to γH and acting on the sites with a normal ν on which rock breaking occurs, $\bar K$ is the rock jointing factor, also referred to γH, and φ is the internal friction angle of the rock. In the following, it is assumed that the massif is isotropic and its rocks have $\bar K = 0.25$ and $\varphi = 20^\circ$ and are in a hydrostatic stress field ($\Lambda = 1$).

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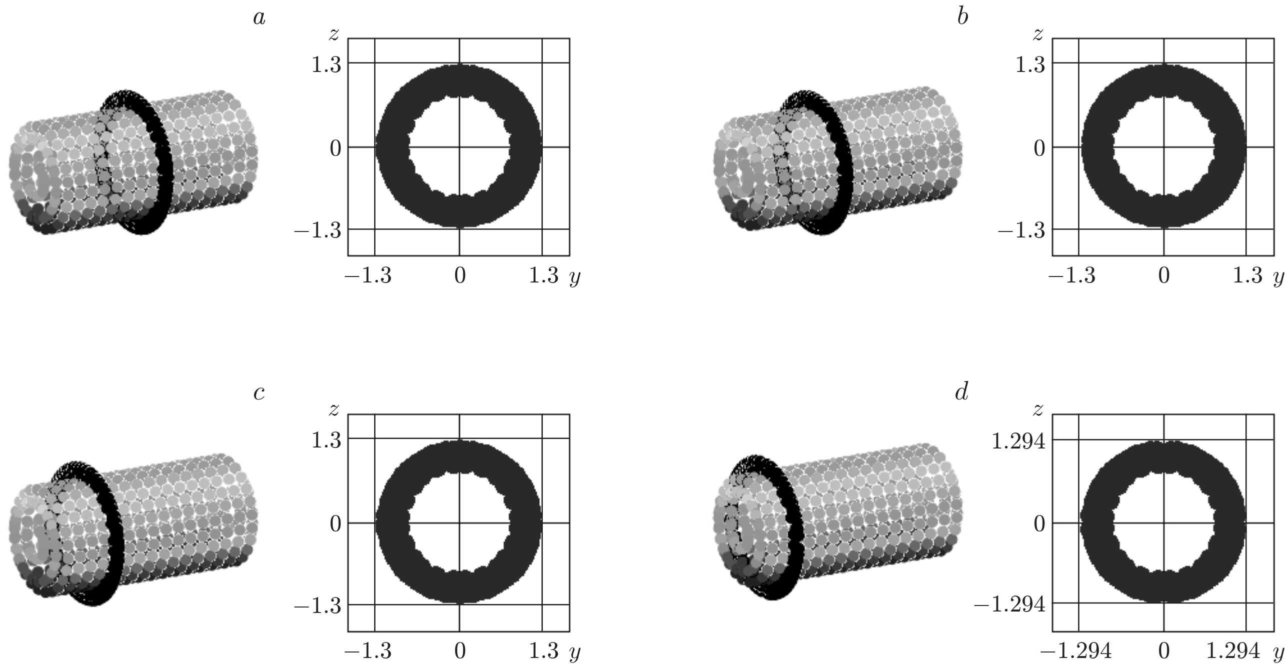

Fig. 1. Discontinuity zone around an unsupported tunnel.

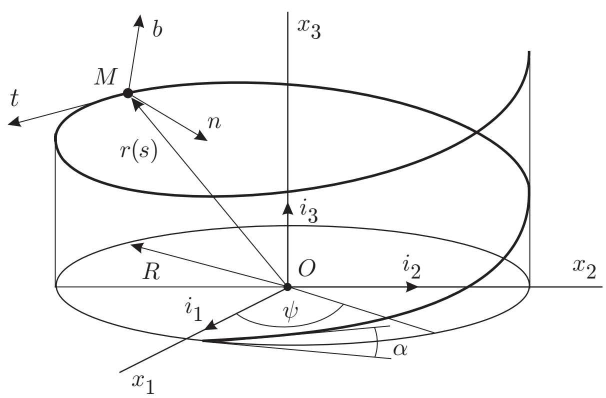

Fig. 2. Geometrical parameters of the cylindrical spring.

Figure 1 shows (in axonometry) a circular tunnel of length L and discontinuity zones (darkened areas) in the cross sections located at distances $0.5L$ (a), $0.375L$ (b), $0.25L$ (c), and $0.125L$ (d) from the tunnel face. It is evident that the discontinuity zones around the unsupported tunnel in this case is a region bounded by the external circular contour and the tunnel contour.

The stability of tunnels is usually increased using various types of support. As a rule, the supports are enclosing or bearing structures. They do not influence the formation of discontinuity zones since they do not produce a reactive response $[F_q(Q_0) = 0]$. In this sense, such supports are passive structures. An anchor-type support can influence the dimensions of the discontinuity zones due by increasing the anchor tension and, hence, it is active. In the present paper, we discuss the possibility of increasing the stability of tunnels of circular cross section using a precompressed cylindrical spring.

For a cylindrical spring (see Fig. 2) made of a rod of circular cross section of length l whose material obeys Hooke's law, the stress–strain state is described by the following system of differential equations in a coupled system of axes [3]:

$$\frac{dQ_1}{ds} - æ_{30}Q_2 - \frac{1}{A_{33}} M_3Q_2 + \frac{1}{A_{22}} M_2\,Q_3 + q_1 = 0; \tag{2a}$$

$$\frac{dQ_2}{ds} + æ_{30}Q_1 - æ_{10}Q_3 + \frac{1}{A_{33}} M_3Q_1 - \frac{1}{A_{11}} M_1Q_3 + q_2 = 0; \tag{2b}$$

$$\frac{dQ_3}{ds} + æ_{10}Q_2 - \frac{1}{A_{22}} M_2Q_1 + \frac{1}{A_{11}} M_1Q_2 + q_3 = 0; \tag{2c}$$

$$\frac{dM_1}{ds} - æ_{30}M_2 = 0; \tag{2d}$$

$$\frac{dM_2}{ds} + æ_{30}M_1 - æ_{10}M_3 + \frac{1}{A_{33}} M_3M_1 - \frac{1}{A_{11}} M_1M_3 - Q_3 = 0; \tag{2e}$$

$$\frac{dM_3}{ds} + æ_{10}M_2 - \frac{1}{A_{22}} M_2M_1 + \frac{1}{A_{11}} M_1M_2 + Q_2 = 0; \tag{2f}$$

$$\frac{d\theta_1}{ds} + \Big(1 - \frac{\cos\theta_2}{\cos\theta_3}\Big)æ_{10} + \Big(\sin\theta_1 \tan\,\theta_3 - \frac{\sin\theta_2}{\cos\theta_3}\Big)æ_{30}$$
$$- \frac{1}{A_{11}} M_1 \frac{\cos\theta_2}{\cos\theta_3} - \frac{1}{A_{33}} M_3 \frac{\sin\theta_2}{\cos\theta_3} = 0; \tag{2g}$$

$$\frac{d\theta_2}{ds} - æ_{10}\cos\theta_2 \tan\,\theta_3 + \Big(\frac{\sin\theta_1}{\cos\theta_3} - \sin\theta_2 \tan\,\theta_3\Big)æ_{30}$$
$$- \frac{1}{A_{11}} M_1 \cos\theta_2 \tan\,\theta_3 - \frac{1}{A_{22}} M_2 - \frac{1}{A_{33}} M_3 \sin\theta_2 \tan\,\theta_3 = 0; \tag{2h}$$

$$\frac{d\theta_3}{ds} + æ_{10}\sin\theta_2 + (\cos\theta_1 - \cos\theta_2)æ_{30} + \frac{1}{A_{11}} M_1 \sin\theta_2 - \frac{1}{A_{33}} M_3 \cos\theta_2 = 0; \tag{2i}$$

$$\frac{du_1}{ds} - æ_{30}u_2 - \frac{1}{A_{33}} M_3u_2 + \frac{1}{A_{22}} M_2u_3 + \cos\theta_2 \cos\theta_3 - 1 = 0; \tag{2j}$$

$$\frac{du_2}{ds} + æ_{30}u_1 - æ_{10}u_3 + \frac{1}{A_{33}} M_3u_1 - \frac{1}{A_{11}} M_1u_3 - \sin\theta_3 = 0; \tag{2k}$$

$$\frac{du_3}{ds} + æ_{10}u_2 - \frac{1}{A_{22}} M_2u_1 + \frac{1}{A_{11}} M_1u_2 + \sin\theta_2 \cos\theta_3 = 0. \tag{2l}$$

Here Q_1, Q_2, and Q_3 are the longitudinal and shearing forces in the cross section of the rod from which the spring is made, M_1, M_2, and M_3 are the torsional and flexural moments in the same cross section of the rod, A_{11}, A_{22}, and A_{33} are the twisting and flexural rigidities of the rod cross section, θ_1, θ_2, and θ_3 are the rotation angles of the axial line of the rod about the rod axis, its main normals, and binormals, respectively, u_1, u_2, and u_3 are the displacements along the rod axis, the main normal, and the binormal, respectively, and q_1, q_2, and q_3 are the components of the external load distributed along the rod length. The torsion $æ_{10}$ and curvatures $æ_{20}$ and $æ_{30}$ of the axial line of the rod in the natural state are defined as follows [4]:

$$æ_{10} = \sin\alpha \cos\alpha / R, \qquad æ_{30} = \cos^2\alpha / R, \qquad æ_{20} = 0. \tag{3}$$

Here R is the radius of the undeformed spring and α is the angle of lead of its coils. Since the spring material operates in the elastic stage, the following formulas are valid:

$$M_1 = A_{11}(æ_1 - æ_{10}), \qquad M_2 = A_{22}(æ_2 - æ_{20}), \qquad M_3 = A_{33}(æ_3 - æ_{30}) \tag{4}$$

($æ_1$, $æ_2$, and $æ_3$ are the torsion and curvature components of the axial line of the deformed rod).

Let a cylindrical shell of circular cross section of radius R be supported by a cylindrical spring of the same radius and is uniformly compressed by the quantity Δ. In this case, the value of the displacement u_2 in the rod is constant and equal to the shell compression Δ. Let the spring ends be not fixed; then, their displacements are obviously symmetric about the point located at the middle of the rod and the middle point itself does not move. Hence, if the coordinate origin is made coincident with this point, the displacements u_1 and u_3 and the rotation angle θ_1 at the coordinate origin are equal to zero:

$$u_1(0) = 0, \qquad u_3(0) = 0, \qquad \theta_1(0) = 0. \tag{5}$$

Since the shell remains a circular cylinder after the uniform compression, the spring inside the shell also retains a cylindrical shape after deformation. Therefore, the rotation angle $\theta_3 = 0$ and Eqs. (2g)–(2l) become

$$\frac{d\theta_1}{ds} + (1 - \cos\theta_2)æ_{10} - æ_{30}\sin\theta_2 - \frac{1}{A_{11}} M_1 \cos\theta_2 - \frac{1}{A_{33}} M_3 \sin\theta_2 = 0; \tag{6a}$$

$$\frac{d\theta_2}{ds} + æ_{30}\sin\theta_1 - \frac{1}{A_{22}} M_2 = 0; \tag{6b}$$

$$æ_{10}\sin\theta_2 + (\cos\theta_1 - \cos\theta_2)æ_{30} + \frac{1}{A_{11}} M_1 \sin\theta_2 - \frac{1}{A_{33}} M_3 \cos\theta_2 = 0; \tag{6c}$$

$$\frac{du_1}{ds} - æ_{30}u_2 - \frac{1}{A_{33}} M_3 u_2 + \frac{1}{A_{22}} M_2 u_3 + \cos\theta_2 - 1 = 0; \tag{6d}$$

$$æ_{30}u_1 - æ_{10}u_3 + \frac{1}{A_{33}} M_3 u_1 - \frac{1}{A_{11}} M_1 u_3 = 0; \tag{6e}$$

$$\frac{du_3}{ds} + æ_{10}u_2 - \frac{1}{A_{22}} M_2 u_1 + \frac{1}{A_{11}} M_1 u_2 + \sin\theta_2 = 0. \tag{6f}$$

By virtue of formulas (4), Eq. (6a) implies the equation

$$\frac{d\theta_1}{ds} + æ_{10} - æ_1 \cos\theta_2 - æ_3 \sin\theta_2 = 0, \tag{7}$$

in which the torsion $æ_1$ and curvature $æ_3$ of the cylindrical spring after its compression can be defined by formulas (3):

$$æ_1 = \sin\alpha_1 \cos\alpha_1 / R_1, \qquad æ_3 = \cos^2\alpha_1 / R_1, \tag{8}$$

where α_1 and $R_1 = R - u_2$ are the angle of lead of the spring coils and the spring radius after the compression. Taking into account formulas (8), we write Eq. (7) as

$$\frac{d\theta_1}{ds} + æ_{10} - \frac{\cos\alpha_1 \sin(\alpha_1 + \theta_2)}{R(1 - \bar{u}_2)} = 0, \tag{9}$$

where $\bar{u}_2 = u_2/R$ is the dimensionless value of the compression.

Next, using formulas (4) and (8), we write Eq. (6c) as

$$æ_{30}\cos\theta_1 - \cos\alpha_1 \cos(\alpha_1 + \theta_2)/(R(1 - \bar{u}_2)) = 0. \tag{10}$$

Simultaneous solution of Eqs. (9) and (10) yields

$$\frac{d\theta_1}{ds} + æ_{10} - æ_{30} \tan\,(\alpha_1 + \theta_2) \cos\theta_1 = 0. \tag{11}$$

We note that after the deformation, the angle of lead of the spring coils α_1 is the algebraic sum of the angle of lead of the coils α of the undeformed spring and the rotation angle θ_2 about the principal normal:

$$\alpha_1 = \alpha - \theta_2. \tag{12}$$

With allowance for formula (12), Eq. (11) becomes

$$\frac{d\theta_1}{ds} - (\cos\theta_1 - 1)æ_{10} = 0.$$

By virtue of the third boundary condition (5) and the solution uniqueness theorem, its solution is trivial:

$$\theta_1 = 0. \tag{13}$$

Equation (10) and formulas (12) and (13) imply the equation

$$\bar{æ}_{30}(1-\bar{u}_2) - \cos(\alpha-\theta_2)\cos\alpha = 0 \tag{14}$$

($\bar{æ}_{30} = æ_{30}R$), from which it follows that $\theta_2 = \text{const}$.

Equation (14) is brought to the form

$$\tan^2\theta_2 - a\tan\theta_2 + b = 0, \tag{15}$$

where

$$a = \frac{2\tan\alpha}{(1-\bar{u}_2)^2 - \tan^2\alpha}, \qquad b = \frac{(1-\bar{u}_2)^2 - 1}{(1-\bar{u}_2)^2 - \tan^2\alpha}.$$

Equation (15) has two roots — $\theta_{2(1)}$ and $\theta_{2(2)}$. One root (negative) corresponds to the right coiling of the spring ($æ_{10} > 0$), and the other (positive) root to the left coiling ($æ_{10} < 0$). Below, we consider the spring with right coiling. Taking into account that $\theta_1 = 0$ and $\theta_2 = \text{const}$, from Eqs. (6a)–(6c) we obtain the following expressions for the moments in dimensionless form:

$$\bar{M}_1 = M_1R/A_{11} = \bar{æ}_{10}\cos\theta_2 - \bar{æ}_{30}\sin\theta_2 - \bar{æ}_{10}, \qquad \bar{M}_2 = 0,$$

$$\bar{M}_3 = M_3R/A_{33} = \bar{æ}_{10}\sin\theta_2 + \bar{æ}_{30}\cos\theta_2 - \bar{æ}_{30}.$$

Here $\bar{æ}_{10} = æ_{10}R$, $\bar{æ}_{30} = æ_{30}R$ is the dimensionless torsion and curvature of the axial line of the spring.

Taking into account that the moments M_1 and M_3 are constant and $M_2 = 0$, from Eqs. (2e) and (2f) we find the internal forces Q_2 and Q_3 in the compressed spring (in dimensionless form):

$$Q_2 = 0, \qquad \bar{Q}_3 = Q_3R^2/A_{11} = \bar{æ}_{30}\bar{M}_1 - m\bar{æ}_{10}\bar{M}_3 + \bar{M}_1\bar{M}_3(1-m)$$

($m = A_{33}/A_{11}$).

Next, writing Eqs. (2a)–(2c) in dimensionless form, we have

$$\frac{d\bar{Q}_1}{d\bar{s}}\bar{\lambda} + \bar{q}_1 = 0; \tag{16a}$$

$$(\bar{æ}_{30} + \bar{M}_3)\bar{Q}_1 - (\bar{æ}_{10} + \bar{M}_1)\bar{Q}_3 + \bar{q}_2 = 0; \tag{16b}$$

$$q_3 = 0 \tag{16c}$$

where $\bar{\lambda} = \cos\alpha$, $\bar{s} = s\bar{\lambda}/R$ is a dimensionless coordinate, and the dimensionless components of the distributed external load are defined as

$$\bar{q}_i = q_iR^3/A_{11}. \tag{17}$$

Since $q_3 = 0$, system (16) reduces to two equations which contain three unknown functions $\bar{Q}_1$, $\bar{q}_1$, and $\bar{q}_2$, and, hence, it is indeterminate. To eliminate this indeterminacy, we use Coulomb's hypothesis

$$\bar{q}_1 = f\bar{q}_2, \tag{18}$$

in which the friction force is the component q_1 and f is the coefficient of rod friction on the shell (below, $f = 0.55$). Since the direction of the friction force q_1 is opposite to the displacement u_1, its value in Eq. (16a) should be negative. In view of the aforesaid, system (16) becomes

$$\frac{d\bar{Q}_1}{d\bar{s}}\bar{\lambda} - f\bar{q}_2 = 0; \tag{19a}$$

$$(\bar{æ}_{30} + \bar{M}_3)\bar{Q}_1 - (\bar{æ}_{10} + \bar{M}_1)\bar{Q}_3 + \bar{q}_2 = 0. \tag{19b}$$

Eliminating $\bar{q}_2$ from (19), we obtain the differential equation

$$\frac{d\bar{Q}_1}{d\bar{s}} + k\bar{Q}_1 - \beta = 0, \tag{20}$$

where

$$k = f(\bar{æ}_{30} + \bar{M}_3)/\bar{\lambda}, \qquad \beta = f(\bar{æ}_{10} + \bar{M}_1)\bar{Q}_3/\bar{\lambda}.$$

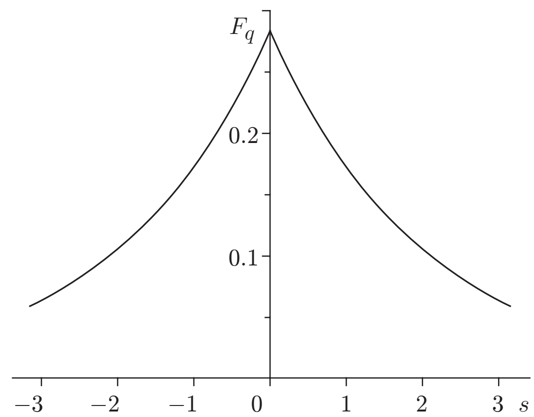

Fig. 3. Distribution of the response $F_q(Q_0)$ along the axis of the spring coil.

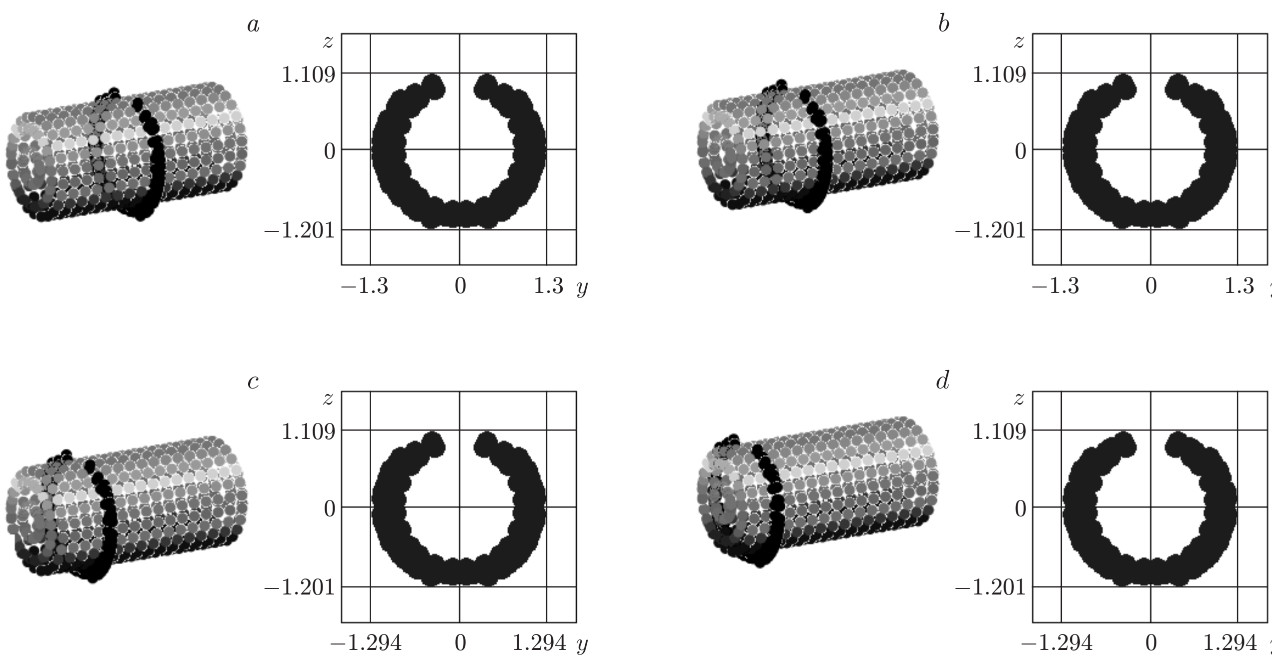

Fig. 4. Discontinuity zone around the supported tunnel.

Since the spring ends are free, the longitudinal force in the end sections is

$$\bar{Q}_1(\bar{l}/2) = 0. \tag{21}$$

Expression (21) is the boundary condition for the differential equation (20), whose solution in this case has the form

$$\bar{Q}_1 = \beta(1 - \mathrm{e}^{-k(\bar{s}-0.5\bar{l})})/k. \tag{22}$$

Next, from Eq. (16a) and formulas (18), we obtain $\bar{q}_1$:

$$\bar{q}_1 = \frac{d\bar{Q}_1}{d\bar{s}}\,\bar{\lambda} \qquad \Longrightarrow \qquad \bar{q}_1 = \bar{\lambda}\beta\,\mathrm{e}^{-k(\bar{s}-0.5\bar{l})}, \tag{23}$$

and from Eq. (19b), the distributed load is

$$\bar{q}_2 = (\bar{æ}_{10} + \bar{M}_1)\bar{Q}_3 - (\bar{æ}_{30} + \bar{M}_3)\bar{Q}_1. \tag{24}$$

Thus, for uniform compression of the cylindrical spring, it should be subjected to a nonuniform external load whose components are defined by formulas (23) and (24). If the uniformly compressed cylindrical spring is placed in a tunnel of circular cross section, the latter produces a reactive response that acts on the surrounding massif and is defined as

$$F_q(Q_0) = q_2/(L\gamma H). \tag{25}$$

Here $L = hn$ is the length of the spring, $h = 2\pi R\tan\alpha$ is the intercoil distance, and n is the number of coils.

In our opinion, it is reasonable to use a spring consisting of several coils joined to one another rather than a continuous spring. The fact is that in a continuous spring, the maximum response (which is greater than that in the coils of a compound spring) arises in its middle. With distance from the middle, the response decreases sharply. In a compound spring, each coil produces the same response whose value is maximal in the middle of the coil and decreases somewhat at its ends. Therefore, each coil of the compound spring needs to be established in such a manner that its ends are in the tunnel floor and the middle is in the roof.

In view of the aforesaid, in formula (25) one need to adopt $L = h$, setting $n = 1$. Next, by virtue of (17), Eq. (25) is brought to the form

$$F_q(Q_0) = 0.1\bar{q}_2\bar{d}^4/(2\pi\overline{\gamma H}\tan\alpha),$$

where $\overline{\gamma H} = \gamma H/E$ (E is Young's modulus of the spring material) and $\bar{d} = d/R$ is the dimensionless diameter of the rod from which the spring is manufactured.

Let a compound spring precompressed by $\bar{u}_2 = 0.1$ and containing eight coils be placed in a tunnel at depth $H = 400$ m in a rock massif with $\gamma = 25$ kN/m^3. The coils of the spring are made of a rod of circular cross section $\bar{d} = 0.1$ with a lead angle $\alpha = 5°$. In this case, each coil of the spring produces a response $F_q(Q_0)$, whose distribution along the coil axis is shown in Fig. 3. The effect of the response of the spring coils on the dimensions of the discontinuity zone is given in Fig. 4, which shows the discontinuity zones in the same sections of the tunnel in which they were previously determined around the unsupported tunnel (see Fig. 1). It is evident that the discontinuity zone has the shape of a horseshoe symmetric about the vertical, whose open ends are located in the tunnel arch, where there is the greatest response of the spring coils. In this case, the discontinuity zone decreases by 14.7% in the tunnel roof and by 7.6% in the floor compared to the discontinuity zone around the unsupported tunnel and the decrease occurs uniformly along the tunnel axis. These results were obtained in the presence of only the force $\bar{q}_2$ since the force $\bar{q}_1$ has almost no effect on the dimension of the discontinuity zone and only breaks its symmetry.

Thus, a precompressed cylindrical spring placed in a tunnel raises its stability and, hence, can be used as a support.

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